**EEE 587 Optimal Control**

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**Discrete Time System**

Text

Description automatically generated with medium confidence

**Determine the optimal control policy which minimizes the performance:**

A picture containing logo

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**Feasible states and controls are defined as follows:**

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**Dynamic Programming Equation**

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**Finding the optimal policy**

Back propagation – Second Stage First (last stage)

K = 2

Cost = C1,2\* (x(1), u(1)) = J1,2 (x(1), u(1)) + J22\* = 2 |x(1) – 0.1 x 12| + 2 |x(2) – 0.1 x 22| + |u(1)|

x(k) = 0, 0.1, 0.2, 0.3, 0.4

u(k) = -0.2, -0.1, 0, 0.1, 0.2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Current State | Control | Next State | Cost | Minimum Cost | Optimal Control Applied |
| x(1) | u(1) | x(2) = x(1) + u(1) | C1,2\* [x(1), u(1)] = 2 |x(1) – 0.1 x 12| + 2 |x(2) – 0.1 x 22 | + |u(1)| | J\* | u \* x(1) at k = 1 |
| 0 | 0 | 0 | 1 | 0.8 | 0.2 |
| 0.1 | 0.1 | 0.9 |
| 0.2 | 0.2 | 0.8 |
| 0.1 | -0.1 | 0 | 0.9 | 0.4 | 0.2 |
| 0 | 0.1 | 0.6 |
| 0.1 | 0.2 | 0.5 |
| 0.2 | 0.3 | 0.4 |
| 0.2 | -0.2 | 0 | 1.2 | 0.4 | 0.2 |
| -0.1 | 0.1 | 0.9 |
| 0 | 0.2 | 0.6 |
| 0.1 | 0.3 | 0.5 |
| 0.2 | 0.4 | 0.4 |
| 0.3 | -0.2 | 0.1 | 1.2 | 0.5 | 0.1 |
| -0.1 | 0.2 | 0.9 |
| 0 | 0.3 | 0.6 |
| 0.1 | 0.4 | 0.5 |
| 0.4 | -0.2 | 0.2 | 1.2 | 0.6 | 0 |
| -0.1 | 0.3 | 0.9 |
| 0 | 0.4 | 0.6 |

First Stage Second

J\*12 (𝑥(1)) stored from previous stage

K = 1

Cost = C0,2\* (x(0), u(0)) = J0,1 (x(0), u(0)) + J12\* = |u(0)| + J12\*(x(1))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Current State | Control | Next State | Cost | Minimum Cost | Optimal Control Applied |
| x(0) | u(0) | x(1) = x(0) + u(0) | C0,2\* [x(0), u(0)] = J12\*(x(1)) + |u(0)| | J\* | u \* x(1) at k = 0 |
| 0 | 0 | 0 | 0.8 | 0.5 | 0.1 |
| 0.1 | 0.1 | 0.5 |
| 0.2 | 0.2 | 0.6 |
| 0.1 | -0.1 | 0 | 0.9 | 0.4 | 0 |
| 0 | 0.1 | 0.4 |
| 0.1 | 0.2 | 0.5 |
| 0.2 | 0.3 | 0.7 |
| 0.2 | -0.2 | 0 | 1 | 0.4 | 0 |
| -0.1 | 0.1 | 0.5 |
| 0 | 0.2 | 0.4 |
| 0.1 | 0.3 | 0.6 |
| 0.2 | 0.4 | 0.8 |
| 0.3 | -0.2 | 0.1 | 0.6 | {0.5, 0.5} | {-0.1, 0} |
| -0.1 | 0.2 | 0.5 |
| 0 | 0.3 | 0.5 |
| 0.1 | 0.4 | 0.7 |
| 0.4 | -0.2 | 0.2 | 0.6 | {0.6, 0.6, 0.6} | {-0.2, -0.1, 0} |
| -0.1 | 0.3 | 0.6 |
| 0 | 0.4 | 0.6 |

**Question 1**

**Determine the optimal control law u\*(𝑥(𝑘), 𝑘)**

**Ans.**

Using the two tables above for the first and second stage, to find the optimal control law u\* we find the action with the least cost for each state x(k)

**Question 2**

**Determine the optimal control sequence {u\*(0), u\*(1)}, if the initial state value is x(0) = 0.2**

**Ans.**

Initial State Value is x(0) is 0.2

We refer to first stage table (k=0) where optimal control action is 0 for x(0) = 0.2

Therefore u\*(0) = 0

Next, referring to second stage table (k=1), optimal control action is 0.2 for x(1) = 0.2

Therefore u\*(1) = 0.2

Hence, Optimal Control Policy is {u\*(0), u\*(1)} = {0, 0.2}